Trigonometric Identities

FUNDAMENTAL IDENTITIES

$$\tan A = \frac{\sin A}{\cos A} \qquad \cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A}$$

$$\sec A = \frac{1}{\cos A} \qquad \csc A = \frac{1}{\sin A}$$

$$\sin(-A) = -\sin A \qquad \cos(-A) = \cos A$$

$$\tan(-A) = -\tan A \qquad \cot(-A) = -\cot A$$

$$\sec(-A) = \sec A \qquad \csc(-A) = -\csc A$$

$$\sin^2 A + \cos^2 A = 1$$

$$\tan^2 A + 1 = \sec^2 A$$

$$1 + \cot^2 A = \csc^2 A$$

SUM AND DIFFERENCE IDENTITIES

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

DOUBLE-ANGLE IDENTITIES

$$\sin 2A \quad 2\sin A\cos A$$

$$\cos 2A \quad \cos^2 A \quad \sin^2 A$$

$$\cos 2A \quad 1 \quad 2\sin^2 A$$

$$\cos 2A \quad 2\cos^2 \quad 1$$

$$\tan 2A \quad \frac{2\tan A}{1 - \tan^2 A}$$

HALF-ANGLE IDENTITIES

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}} \qquad \tan\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}} \qquad \tan\frac{A}{2} = \frac{\sin A}{1+\cos A}$$

$$\tan\frac{A}{2} = \frac{1-\cos A}{\sin A}$$

COFUNCTION IDENTITIES

$\sin(90 \quad A) \quad \cos A \quad \sin(- \quad A) \quad \cos A$

$\cos(90 \quad A) \quad \sin A \quad \cos(- \quad A) \quad \sin A$

$\tan(90 \quad A) \quad \cot A \quad \tan(\frac{}{2} \quad A) \quad \cot A$

$\cot(90 \quad A) \quad \tan A \quad \cot(\frac{}{2} \quad A) \quad \tan A$

$\sec(90 \quad A) \quad \csc A \quad \sec(\frac{}{2} \quad A) \quad \csc A$

$\csc(90 \quad A) \quad \sec A \quad \csc(\frac{}{2} \quad A) \quad \sec A$

SOLVING TRIANGLES

Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

SUM-TO-PRODUCT AND PRODUCT-TO-SUM

$$\sin A\sin B = \tfrac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$\sin A\cos B = \tfrac{1}{2}[\sin(A+B) + \sin(A-B)]$$

$$\cos A\cos B = \tfrac{1}{2}[\cos(A+B) + \cos(A-B)]$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\sin^2 A = \frac{1-\cos 2A}{2}$$

$$\cos^2 A = \frac{1+\cos 2A}{2}$$